

1. Perform the following additions.

- a)  $7 + -4 = \underline{3}$     b)  $-6 + -7 = \underline{-13}$     c)  $-4 + 9 = \underline{5}$     d)  $8 + -5 = \underline{3}$   
 e)  $-4 + -4 = \underline{-8}$     f)  $4 + -15 = \underline{-11}$     g)  $6 + -6 = \underline{0}$     h)  $-12 + 9 = \underline{-3}$   
 i)  $8 + 7 = \underline{15}$     j)  $-6 + -10 = \underline{-16}$     k)  $-14 + 24 = \underline{10}$     l)  $-9 + -15 = \underline{-24}$

2. Perform the following additions.

- a)  $-4 + -3 + 9 = \underline{2}$     b)  $-7 + 12 + -6 = \underline{-1}$     c)  $-5 + -6 + -4 = \underline{-15}$   
 d)  $8 + -3 + 12 + -4 = \underline{13}$     e)  $-7 + -12 + 18 = \underline{-1}$     f)  $8 + -9 + -7 + -4 = \underline{-12}$   
 g)  $6 + -9 + -7 + 10 = \underline{0}$     h)  $5 + -7 + -3 + 7 = \underline{2}$     i)  $7 + -4 + -3 + -11 = \underline{-11}$

3. Estimate the following sums. Then calculate them.

- a)  $-100 + -35 = \underline{-135}$     b)  $-140 + 128 = \underline{-12}$     c)  $-346 + -132 = \underline{-478}$   
 d)  $-245 + 348 = \underline{103}$     e)  $156 + -245 = \underline{-89}$     f)  $48 + -212 = \underline{-164}$

4. Consider the two tables below:

Table 1

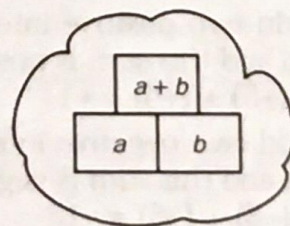
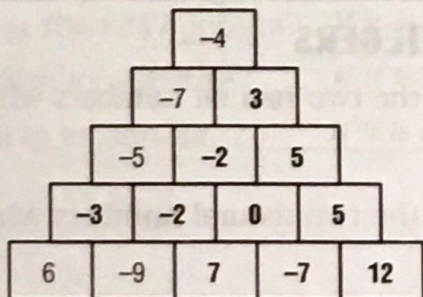
-5	-8	7	-2	-1	-9
-4	6	-8	4	-5	-7
3	-2	-7	2	-1	-5
5	-6	-1	-9	8	-3
-3	8	5	-4	9	15
-4	-2	-4	-9	10	-9

Table 2

6	5	-4	3	8	18
-7	4	5	-2	-6	-6
-5	-2	-8	5	6	-4
8	-1	-4	-8	-3	-8
9	0	7	-2	1	15
11	6	-4	-4	6	15

- a) Determine the sum of the integers in each row and write this sum in the box at the right of the row.  
 b) Determine the sum of the integers in each column and write this sum in the box at the bottom of the column.  
 c) Check that the sum of the results found in a) is equal to the sum of the results found in b).

5. Complete the following pyramid, given that each number written in a rectangle is equal to the sum of the two integers in the rectangles which support it.



**6.** Determine the value of  $a$  in each of the following cases.

- a)  $a + -7 = -4$   $a = 3$     b)  $-6 + a = 0$   $a = 6$     c)  $4 + a = 2$   $a = -2$   
 d)  $-4 + a = -7$   $a = -3$     e)  $5 + a = -8$   $a = -13$     f)  $-6 + a = 11$   $a = 17$   
 g)  $12 + a = 0$   $a = -12$     h)  $-4 + a = 2$   $a = 6$     i)  $a + -6 = -7$   $a = -1$

**7.** If  $a = -14$  and  $b = 6$ , determine the numerical value of the following sums.

- a)  $a + b = -8$     b)  $b + a = -8$

What can be said about the addition of integers? *It is commutative.*

**8.** Given that  $a = -6$ ,  $b = 8$  and  $c = -9$ , determine the numerical value of the following sums.

- a)  $(a + b) + c = 2 + -9 = -7$     b)  $a + (b + c) = -6 + -1 = -7$

What can be said about the addition of integers? *It is associative.*

**9.** Verify the properties of addition in  $\mathbb{Z}$  using integers of your choice.

- a) The sum of integers is also an integer. *Various answers possible*  
 b) The operation of addition is commutative. *Various answers possible*  
 c) The operation of addition is associative. *Various answers possible*  
 d) 0 is the neutral element of addition in  $\mathbb{Z}$ . *Various answers possible*  
 e) The sum of two opposite integers is zero. *Various answers possible*

**10.** Is it possible to find two integers whose sum is 0 if these two numbers are:

- a) positive? *No*  
 b) negative? *No*  
 c) of opposite signs? *Yes, for example -8 and +8.*

**11.** Find, if possible, two consecutive integers whose sum is:

- a)  $-7$ : *-4 and -3*    b)  $9$ : *4 and 5*    c)  $-1$ : *-1 and 0*  
 d)  $0$ : *Not possible*    e)  $-4$ : *Not possible*    f)  $-25$ : *-13 and -12*

**12.** Consider the integers  $a = -6$  and  $b = 9$ . Determine:

- a) The opposite of the sum of these two numbers. *-3*  
 b) The sum of the opposite of  $a$  and the opposite of  $b$ . *6 + -9 = -3*  
 c) Insert the appropriate symbol below ( $=$  or  $\neq$ ).  
 $\text{opp}(a + b) \boxed{=} \text{opp}(a) + \text{opp}(b)$

**13.** Complete the following tables in order to obtain magic squares. (The sum of the numbers in each row, column and diagonal is always the same.)

3	-2	-1
-4	0	4
1	2	-3

-5	0	-1
6	-2	-6
-3	-4	1

-5	0	-7
-6	-4	-2
-1	-8	-3

- 14** Cleopatra was born in 69 B.C. She died at the age of 39.

In what year did she die? She died in the year 30 BC.

- 15** The month of February, 2003, was very cold in Montreal. On Saturday, the 15<sup>th</sup> of February, the temperature was  $-25^{\circ}\text{C}$ . During the following week, the temperature rose  $19^{\circ}\text{C}$  and then fell again  $8^{\circ}\text{C}$  on the weekend. What was the temperature that weekend?

It was  $-14^{\circ}\text{C}$ .

- 16** Peter works as a delivery boy in an office building. He begins working on the 18<sup>th</sup> floor, descends 9 floors, ascends 2 floors, descends 13, ascends 4, descends 5 and finally ascends 8 more floors to go to eat at the building's cafeteria. On what floor is the cafeteria located?

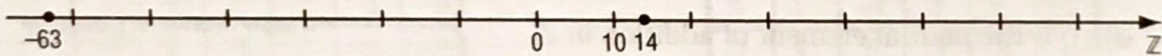
The cafeteria is on the 5<sup>th</sup> floor.

### ACTIVITY 3 Subtraction of integers

The Roman emperor Augusta was born in 63 B.C. and died in 14 A.D..

- a) Represent these two numbers on the number line below and determine the number of years that he lived.

He lived for 77 years.



- b) 1. Write the subtraction which should be used to determine the number of years Augusta lived.  $14 - (-63)$

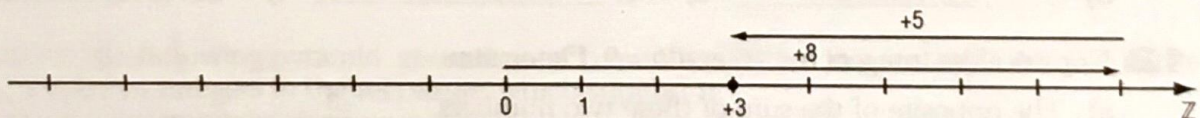
2. Transform this subtraction into an addition and verify the result obtained in a).

$14 + 63 = 77$ . He lived for 77 years.

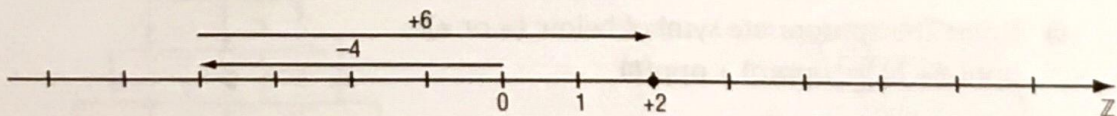
### ACTIVITY 4 The number line

- a) With the help of the number line, perform the following subtractions.

1.  $(+8) - (+5) = (+3)$



2.  $(-4) - (+6) = (+2)$



- b) 1. Subtracting a positive number results in a move towards the left.
2. Subtracting a negative number results in a move towards the right.

## SUBTRACTION OF INTEGERS

Subtracting an integer is the same as adding its opposite.

$$a - b = a + \text{opp}(b)$$

Ex.:  $3 - (-5) = 3 + 5 = 8$

$-4 - 5 = -4 + -5 = -9$

**17.** Transform each of the following subtractions into an addition and calculate the result.

a)  $-4 - 9 = -13$       b)  $5 - 9 = -4$       c)  $-8 - -5 = -3$

d)  $7 - 12 = -5$       e)  $-7 - 8 = -15$       f)  $9 - -12 = 21$

**18.** Perform the following calculations.

a)  $-8 - 12 = -20$       b)  $7 - 9 = -2$       c)  $12 - -14 = 26$

d)  $-15 + 15 = 0$       e)  $-4 + 18 = 14$       f)  $-5 - 17 = -22$

**19.** Perform the following calculations.

a)  $-7 + 12 - 14 - 9 = -18$       b)  $-7 + 4 - 9 + 14 = 2$       c)  $6 - 15 - 3 + 34 = 22$

d)  $-5 + 13 - 9 - 7 = -8$       e)  $5 - 17 + 5 - 21 = -28$       f)  $3 - 12 - 7 + 4 = -12$

**20.** Perform the following calculations.

a)  $(3 - 9) + (-5 - 4) = -15$       b)  $(-7 + 4) - (2 - 9) = 4$       c)  $(-7 + 4 - 12) - (7 - 2) = -20$

d)  $(-8 + 4) - (9 - 3) = -10$       e)  $(-8 + 15) + (5 - 9) = 5$       f)  $(-6 + 5) - (-7 - 5) = 11$

**21.** Perform the following calculations.

a)  $(7 - 9 + 12) + (-5 + 3 - 12) = -4$       b)  $(-6 + 18 - 25) - (4 - 9 - 5) = 3$

c)  $(2 - 8) - (34 - 25 - 7) = -8$       d)  $(-5 - 6 + 9) + (-7 - 11 + 12) = -8$

e)  $9 - (4 - 6 + 2) + (-9 + 2) = 2$       f)  $(-6 + 3 - 9) - (2 - 7 - 8) = 1$

**22.** a) Determine the value of each of the following expressions if  $a = -7$  and  $b = 4$ .

1.  $a - b = -11$       2.  $b - a = 11$

b) Compare the result of the difference  $a - b$  to the difference  $b - a$ .

Verify your answer using other integers. *They are opposites.*

**23.** Determine the value of each of the following expressions if  $a = -4$ ,  $b = 9$  and  $c = -12$ .

a)  $(a - b) - c = -13 + 12 = -1$       b)  $a - (b - c) = -4 - 21 = -25$

**24.** Determine if each of the following statements is true or false. If the statement is false, give a counter-example.

a) The difference of two integers is an integer. True

b) The operation of subtraction is commutative. False

c) The operation of subtraction is associative. False

d) The difference of two opposite integers is zero. False

e) The difference of two positive integers is positive. False

f) If the difference of two integers is zero, then they are opposites. False

- 25.** Determine the value of each of the following expressions if  $a = -7$ ,  $b = 4$  and  $c = -12$ .
- a)  $a + b - c =$  9    b)  $a - b + c =$  -23    c)  $b - a - c =$  23  
d)  $(a - b) + (b - c) =$  5    e)  $(a - b) - (b - c) =$  -27    f)  $(b - c) - a =$  23
- 26.** Find the value of  $x$  in each of the following cases.
- a)  $x - 7 = 8$   $x = 15$     b)  $x - 3 = -7$   $x = -4$     c)  $5 - x = -6$   $x = 11$   
d)  $-7 - x = 8$   $x = -15$     e)  $x - 7 = -2$   $x = 5$     f)  $x - 12 = 0$   $x = 12$
- 27.** Find two integers  $x$  and  $y$  whose sum is  $S$  and whose difference is  $D$ .
- a)  $S = -1$  and  $D = 15$  7 and -8 Other answers possible.    b)  $S = 19$  and  $D = -5$  7 and 12 Other answers possible  
c)  $S = 5$  and  $D = 13$  9 and -4 Other answers possible.    d)  $S = -14$  and  $D = -2$  -8 and -6 Other answers possible.

- 28.** Complete the following tables.

Table 1

$a$	$b$	$a + b$	$a - b$
2	5	7	-3
-7	-8	-15	1
5	-9	-4	14
-7	4	-3	-11

Table 2

$a$	$b$	$a = b$	$a - b$
8	-17	-9	25
-16	-11	-27	-5
-13	6	-7	-19
-9	-13	-22	4

- 29.** How long did each of the following people live?
- a) Plato (-428, -348) 80 years    b) Caesar (-101, -44) 57 years  
c) Antoine (-83, -30) 53 years    d) Alexandre the Great (-356, -323) 33 years
- 30.** Antoine died in the year 30 BC at the age of 53 years. In what year was he born?  
He was born in 83 BC.
- 31.** King David was born in 1015 BC and died in 970 BC. The Roman emperor Augusta was born in 63 BC and died in 14 AD. Which of these two men lived longer?  
Augusta lived longer.
- 32.** Queen Celeste was born in 1015 BC and died at the age of 48. Emperor Romanus died in 212 BC at the age of 54. For how many years were they both alive at the same time?  
For 22 years.
- 33.** There are huge differences in temperature between the Great Quebec North and the St. Laurence Valley. On a given day, it was  $-48^{\circ}\text{C}$  in the Great North and  $7^{\circ}\text{C}$  in Montreal. What is the difference between these two temperatures?  
The difference is  $55^{\circ}\text{C}$ .
- 34.** On a particular winter day at Three Rivers, it was  $2^{\circ}\text{C}$  during the day and  $-7^{\circ}\text{C}$  at night. On the same day in Montreal, the daytime temperature was  $6^{\circ}\text{C}$  and the overnight temperature was  $-5^{\circ}\text{C}$ .
- a) Which city had the greater range in temperature? Montreal  
b) What is the difference between the ranges in temperature?  $2^{\circ}\text{C}$
- 35.** Altitudes and depths are measured with respect to sea level. Sea level corresponds to an altitude of 0. Determine the distance which separates a plane flying at an altitude of 3 250 m and a submarine travelling at a depth of 326 m below sea level.  
3576 m