

## 2.3 Multiplication and division of integers

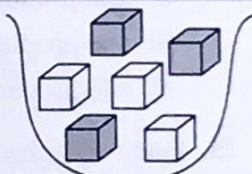
### ACTIVITY 1 Thermal cubes

In a tureen, there is an equal number of blue cubes and white cubes.

A blue cube gives off heat and has the effect of raising the temperature by 1 °C.

A white cube emits cold and has the effect of lowering the temperature by 1 °C.

Complete the following sentences and give the result of each multiplication.



- a) Adding 2 groups of 3 blue cubes results in raising the temperature by 6 °C.

$$(+2) \times (+3) = (+6)$$

- b) Removing 2 groups of 3 blue cubes results in lowering the temperature by 6 °C.

$$(-2) \times (+3) = (-6)$$

- c) Adding 2 groups of 3 white cubes results in lowering the temperature by 6 °C.

$$(+2) \times (-3) = (-6)$$

- d) Removing 2 groups of 3 white cubes results in raising the temperature by 6 °C.

$$(-2) \times (-3) = (+6)$$

### MULTIPLICATION OF INTEGERS

To multiply two integers, we find the product of the two natural numbers which comprise them.

If the two numbers have the same sign, then the product is positive.

If the two numbers have opposite signs, then the product is negative.

Ex.:  $(+5) \times (+8) = +40$        $(-6) \times (-7) = +42$

$(-7) \times (+4) = -28$        $(+5) \times (-9) = -45$

1. Perform the following multiplications.

a)  $-7 \times 5 = -35$       b)  $8 \times 6 = 48$       c)  $-3 \times -4 = 12$       d)  $3 \times -12 = -36$

e)  $-6 \times 0 = 0$       f)  $1 \times -4 = -4$       g)  $-4 \times -4 = 16$       h)  $-1 \times -1 = 1$

2. Complete the two tables below.

Table 1

$\times$	-8	7	-2	-9
-4	32	-28	8	36
3	-24	21	-6	-27
5	-40	35	-10	-45
-2	16	-14	4	18

Table 2

$\times$	5	4	3	3
-7	-35	-28	-21	-21
-5	-25	-20	-15	-15
-6	-30	-24	-18	-18
8	40	32	24	24



3 Find the value of  $a$  in each of the following cases.

- a)  $a \times -5 = 60$   $a = -12$       b)  $7 \times a = -42$   $a = -6$   
 c)  $-9 \times a = 0$   $a = 0$       d)  $-8 \times a = -72$   $a = 9$

4 What can be said about the signs of two integers  $a$  and  $b$  if:

- a) the product  $a \times b$  is positive and the sum is positive?  $a$  and  $b$  are positive.  
 b) the product  $a \times b$  is positive and the sum is negative?  $a$  and  $b$  are negative.  
 c) the product  $a \times b$  is negative?  $a$  and  $b$  are of opposite signs.

5 Find two integers whose product is  $P$  and whose sum is  $S$ .

- a)  $P = -12$  and  $S = -1$   $-4$  et  $3$       b)  $P = -40$  and  $S = -3$   $-8$  et  $5$   
 c)  $P = 28$  and  $S = -11$   $-4$  et  $-7$       d)  $P = -30$  and  $S = 7$   $-3$  et  $10$

### MULTIPLICATION OF MORE THAN 2 INTEGERS

The product of more than 2 integers is:

- positive if the multiplication is made up of an even number of negative factors.  
Ex.:  $-3 \times -4 \times 2 \times -3 \times 2 \times -1 = 144$
- negative if the multiplication is made up of an odd number of negative factors.  
Ex.:  $-2 \times -1 \times 3 \times -1 \times 2 \times -2 \times -1 = -24$
- zero if at least one of the factors is zero.  
Ex.:  $-4 \times 3 \times -2 \times 0 \times -1 \times -2 = 0$

6 Perform the following multiplications.

- a)  $-2 \times 3 \times -2 \times 3 \times -1 \times -5 = 180$       b)  $-1 \times 3 \times -2 \times 2 \times -4 = -48$   
 c)  $3 \times -2 \times 4 \times 0 \times -2 = 0$       d)  $-1 \times 3 \times -2 \times 4 \times -5 \times -1 \times -3 = -360$

7. Consider the following two tables.

Table 1

-1	-2	3	-1	-4	24
-4	1	-2	-1	-5	40
3	-2	-2	2	-1	-24
2	-3	-1	-3	1	-18
-3	-1	1	-4	2	-24
-72	12	-12	24	-40	-9 953 280

Table 2

2	3	-1	3	5	-90
-5	4	2	-2	-1	-80
-1	-2	-1	2	4	-16
6	-1	-4	-1	-3	72
-3	0	-7	-1	1	0
-180	0	56	-12	60	0

- a) Determine the product of the integers in each row and write this product in the box at the right of each row.  
 b) Determine the product of the integers in each column and write this product in the box at the bottom of each column.  
 c) Verify that the product of the results obtained in a) is equal to the product of the results obtained in b).



**8** Fill in the three missing terms in the sequences of integers below.

a) 3, -6, 12, -24,  $\boxed{48}$ ,  $\boxed{-96}$ ,  $\boxed{192}$

b) 5,  $\boxed{-15}$ , 45,  $\boxed{-135}$ , 405,  $\boxed{-1\ 215}$

c) -7, -21, -15, -45, -39,  $\boxed{-117}$ ,  $\boxed{-111}$ ,  $\boxed{-333}$

**9** Determine if each of the following statements is true or false. If the statement is false, give a counter-example.

a) The product of an integer and -1 is the opposite of this integer. True

b) The product of a non-zero integer and its opposite is negative. True

c) The product of two opposites is zero. False

d) If one of the factors of a product is zero, then the product is zero. True

e) If the product of factors is zero, then all the factors are zero. False

**10** Determine the value of  $a$  in each case below.

a)  $-5 \times a = 0$   $a = 0$  b)  $3 \times (a + 3) = 0$   $a = -3$

c)  $(a - 5) \times 4 = 0$   $a = 5$  d)  $2 \times (a + 6) = 0$   $a = -6$

**11.** If  $a = -5$  and  $b = 7$ , determine the numerical value of the following products.

a)  $a \times b =$   $-35$  b)  $b \times a =$   $-35$

What can be said about multiplication of integers? It is commutative.

**12** If  $a = -4$ ,  $b = 6$  and  $c = -2$ , determine the numerical value of the following products.

a)  $(a \times b) \times c =$   $-24 \times -2 = 48$  b)  $a \times (b \times c) =$   $-4 \times -12 = 48$

What can be said about multiplication of integers? It is associative.

**13** Verify the following properties of multiplication in  $\mathbb{Z}$ , using integers of your choice.

a) The product of integers is an integer. Various answers possible.

b) The operation of multiplication is commutative. Various answers possible.

c) The operation of multiplication is associative. Various answers possible.

d) 1 is the neutral element of multiplication. Various answers possible.

e) 0 is the absorption element of multiplication. Various answers possible.

f) Multiplication is an operation which is distributive over addition and subtraction. Various answers possible

**14** Calculate the following, using two different methods.

a)  $-5 \times (-8 + 3) =$   $-5 \times -5 = 25$  b)  $4 \times (5 - 12) =$   $4 \times -7 = -28$   
 $= 40 - 15 = 25$   $= 20 - 48 = -28$

c)  $-7 \times (-8 + 12) =$   $-7 \times 4 = -28$  d)  $3 \times (-15 + 14) =$   $3 \times -1 = -3$   
 $= 56 - 84 = -28$   $= -45 + 42 = -3$

e)  $-3 \times (1 - 7 + 3) =$   $-3 \times -3 = 9$  f)  $5 \times (-6 + 2 - 14) =$   $5 \times -18 = -90$   
 $= -3 + 21 - 9 = 9$   $= -30 + 10 - 70 = -90$



- 15.** Determine the greatest common factor of the terms in each of the following sums and factor it out, rewriting each expression as a product.

a)  $18 - 12 = \underline{6 \times (3 - 2)}$       b)  $-14 - 21 = \underline{-7 \times (2 - 3)}$   
 c)  $9 - 27 - 36 = \underline{9 \times (1 - 3 - 4)}$       d)  $-25 + 15 - 40 = \underline{-5 \times (5 - 3 + 8)}$   
 e)  $4 \times a + 12 = \underline{4 \times (a + 3)}$       f)  $14 \times m - 21 = \underline{7 \times (2m - 3)}$   
 g)  $-9 \times p + 54 = \underline{-9 \times (p - 6)}$       h)  $12 \times r - 24 = \underline{12 \times (r - 2)}$

## ACTIVITY 2 Powers

- a) Complete the table below.

	base	exponent	value
$5^2$	5	2	25
$(-5)^2$	-5	2	25
$2^3$	2	3	8
$(-2)^3$	-2	3	-8

- b) If  $a$  is an integer and  $n$  is a natural number greater than 1, determine the sign of  $a^n$  given that:
- $a$  is positive and  $n$  is even: positive
  - $a$  is negative and  $n$  is even: positive
  - $a$  is positive and  $n$  is odd: positive
  - $a$  is negative and  $n$  is odd: negative
- c) What can be said about the base  $a$  and the exponent  $n$  when  $a^n$  is negative?  
The base is negative and the exponent is odd.

### POWER OF AN INTEGER

The power  $a^n$  is negative when the base  $a$  is negative and the exponent is odd. In all other cases, it is positive.

Ex.:  $3^2 = 9$        $2^3 = 8$        $(-5)^2 = 25$        $(-4)^3 = -64$

**Careful!**  $(-2)^4 = 16$  and  $-2^4 = -16$

- 16.** Determine the value of the following powers.

a)  $(-2)^3 = \underline{-8}$       b)  $(-6)^2 = \underline{36}$       c)  $4^2 = \underline{16}$       d)  $(-8)^0 = \underline{1}$   
 e)  $(-3)^2 = \underline{9}$       f)  $-3^2 = \underline{-9}$       g)  $-3^5 = \underline{-243}$       h)  $(-3)^5 = \underline{-243}$

- 17.** Determine the value of  $a$  in each of the following cases.

a)  $a^3 = -8$   $a = -2$       b)  $2^a = 32$   $a = 5$       c)  $a^3 = -27$   $a = -3$       d)  $(-6)^a = 36$   $a = 2$

- 18.** Perform the following calculations.

a)  $-2 \times 3^2 = \underline{-18}$       b)  $(-4)^2 \times -3 = \underline{-48}$       c)  $5 \times (-2)^3 = \underline{-40}$       d)  $(-2 - 3)^2 = \underline{25}$   
 e)  $-5 \times (-2)^5 = \underline{160}$       f)  $(-3)^4 \times (-2)^3 = \underline{-648}$       g)  $(-3 \times 4)^2 = \underline{144}$       h)  $3 \times (-2)^5 = \underline{-96}$



## DIVISION OF INTEGERS

- The quotient of two integers of the same sign, if it exists, is positive.  
Ex.:  $(+24) \div (+3) = +8$   $(-54) \div (-6) = 9$
- The quotient of two integers of opposite signs, if it exists, is negative.  
Ex.:  $(-45) \div (+5) = -9$   $(+12) \div (-4) = -3$

**19.** Perform the following divisions.

a)  $(-55) \div (+5) = -11$  b)  $(-48) \div (-6) = 8$  c)  $(+54) \div (-9) = -6$  d)  $(+56) \div (+8) = 7$

**20.** Find the value of  $a$  in each of the following cases.

a)  $a \div (-5) = 3$   $a = -15$  b)  $(-63) \div a = 7$   $a = -9$   
c)  $(-45) \div a = -5$   $a = 9$  d)  $a \div (-5) = -1$   $a = 5$

**21.** Find the value, if it exists, of the following expressions, given that  $a = -24$ ,  $b = -6$  and  $c = 2$ .

a)  $a \div b = 4$  b)  $b \div a = \text{Non existant}$   
c)  $(a \div b) \div c = 4 \div 2 = 2$  d)  $a \div (b \div c) = -24 \div -3 = 8$

**22.** Determine if each of the following statements is true or false. If a statement is false, give a counter-example.

- a) The quotient of two integers is an integer. *False*  
b) The operation of the division of integers is commutative. *False*  
c) The operation of division of integers is associative. *False*  
d) The quotient of 0 divided by a non-zero integer is 0. *True*

**23.** Complete the following tables.

**Table 1**

$a$	$b$	$a \times b$	$a \div b$
20	5	100	4
-16	-8	128	2
45	-9	-405	-5
-24	4	-96	-6

**Table 2**

$a$	$b$	$a \times b$	$a \div b$
18	-2	-36	-9
55	-11	-605	-5
-12	4	-48	-3
-24	-6	144	4

**24.** The adjacent table shows the temperature at 6 am every day of a particular winter week.

- a) What was the average temperature at 6 am that week?  
 $-3^\circ\text{C}$
- b) On which days of the week was the temperature less than this average?  
*Monday, Tuesday, Sunday*

Day	Temperature
Monday	$-5^\circ\text{C}$
Tuesday	$-12^\circ\text{C}$
Wednesday	$3^\circ\text{C}$
Thursday	$-2^\circ\text{C}$
Friday	$4^\circ\text{C}$
Saturday	$-1^\circ\text{C}$
Sunday	$-8^\circ\text{C}$

**25.** The adjacent table shows a flea market vendor's daily balance for one week. Determine the balance on Friday if the vendor realized an average profit of \$80 per day that week.

*The balance on Friday was a profit of \$128.*

Day	Balance
Monday	-\$45
Tuesday	-\$72
Wednesday	\$124
Thursday	\$78
Friday	x
Saturday	\$172
Sunday	\$95